New Interpolation Method for Quadrature Encoder Signals

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***Abstract*—This paper presents a new interpolation method suitable for increasing the measurement resolution obtainable from quadrature encoder signals. Based on the existing sinusoidal signals, high-order sinusoids can be derived, from which binary pulses can be generated, which can be decoded using only standard servo controllers for position information. A look-up table, constructed off-line, serves as the inferencing engine for the proposed method. Imperfections in the encoder signals can be directly compensated for in the look-up table, including mean and phase offsets, amplitude difference, and waveform distortion. Simulation and experimental results are provided in this paper.**

***Index Terms*—Encoder resolution, interpolation errors, look-up tables, measurement interpolation, quadrature encoder signals.**

I. Introduction

HIGH-PRECISION and resolution motion control relies critically on the precision and resolution achievable from the encoders. These factors are in turn limited by the technology behind the manufacturing of encoders. To date, the scale grating on linear optical encoders can be manufactured to less than four micrometers in pitch, but, clearly, further reduction in pitch is greatly constrained by physical considerations. This implies an optical resolution of one micrometer is currently achievable. Interpolation using soft techniques will provide an interesting possibility to further improve on the encoder resolution, by processing the analog encoder signals online to yield the small intermediate positions.

The error sources associated with position information obtained this way can be classified under pitch and interpolation errors. Pitch errors are due to scale manufacturing tolerances and mounting distortion. They can be compensated via the same procedures that will be carried out for general geometrical error compensation. Interpolation errors are associated with the accuracy of subdivision within a pitch, affecting any calibration performed. Ideal signals from encoders are a pair of sinusoids with a quadrature phase difference between them. Interpolation operates on the relative difference in amplitude and phase of these paired sinusoids. Therefore, interpolation errors will occur if the pair-periodic signals deviate from the ideal waveforms on which the interpolation computations are based. These deviations must be corrected before interpolation, using digital signal processing techniques, to reduce the interpolation errors.

The technology to compensate the mean value errors, phase and amplitude errors for two quadrature sinusoidal signals was first introduced by Heydemann [1]. He used least squares fitting to compute these error components efficiently and made correction for the two nonideal sinusoidal signals. Using this method, K. P. Birch [2] was able to calculate optical fringe fractions to nanometric accuracy. By making use of the amplitude variation with angle, Birch divided one period of the sinusoidal signal into N equiangular segments to increase the effective electrical angle resolution. The micro step controller [3] and encoder code compensation technology [4] have been developed based on this method. Relevant applications can also be found in [5], [6].

These interpolation approaches generally require explicit high-precision analog-to-digital converters in the control system, and a high-speed DSP to compute the electrical angle to the required resolution. Therefore, they are inapplicable to the typical servo controller with only digital incremental encoder interface. Furthermore, it is cumbersome to integrate sinusoidal correction with interpolation since the correction parameters must be calibrated off-line. As a result, most servo controllers that are able to offer interpolation have assumed perfect quadrature sinusoids. As a result, specifications relating to resolution may be achievable, but the accompanying accuracy cannot be guaranteed. The current effort for sinusoid correction also does not consider error in the form of waveform distortion, i.e., the actual signal may be periodic but is not perfectly sinusoidal. These errors are certainly significant when sub-micron resolution and accuracy are required.

This paper presents a new method to carry out both correction and interpolation, independent of the servo controller. As a result, the method is applicable to most servo controllers, including those with only digital incremental encoder interfaces. The basic idea is to derive high-order sinusoids based on existing quadrature sinusoids from the encoder. These high-order signals may in turn be converted to a series of high-frequency binary pulses that are readily decoded by standard servo controllers. A look-up table is used to implement the idea with little computational requirements, compared to the online computation of electrical angle necessary in current interpolators. Sinusoidal corrections, including mean and phase offsets, amplitude difference and waveform distortion, can be directly reflected in the look-up table. This process is usually done offline, although the table can also be updated adaptively online to reflect any subsequent changes or drift in the encoder signals. Simulation and experimental results are provided to highlight the principles and applicability of the proposed method.

II. Principle of the Proposed Interpolation Method

The basic idea of the proposed interpolation method is to derive high-order sinusoids based on the fundamental one. From

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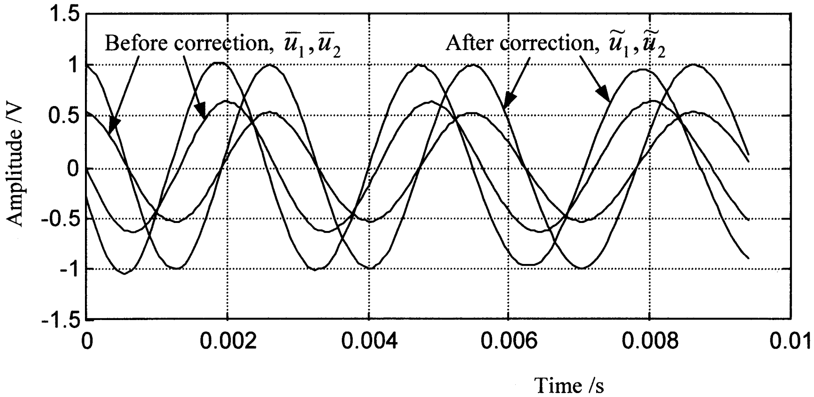

Fig. 1. Sinusoidal signals correction.

these, binary pulses can be generated which can be readily decoded by standard servo controllers for position information. As an example, given the values of $\sin(\alpha)$ and $\cos(\alpha)$, $\sin(2\alpha)$ and $\cos(2\alpha)$ can be obtained from the trigonometry relations:

$$\begin{cases} \sin(2\alpha) = 2\sin(\alpha)\cos(\alpha) \\ \cos(2\alpha) = 1 - 2\sin^2(\alpha). \end{cases} \tag{1}$$

In general, assuming $\sin(\alpha)$ and $\cos(\alpha)$ are known with sufficient precision, $\sin(n\alpha)$ and $\cos(n\alpha)$ $(n \in Z > 1)$ can be derived from the following general equations

$$\begin{cases} \sin(n\alpha) = n\cos^{n-1}(\alpha)\sin(\alpha) - C_n^3\cos^{n-3}(\alpha)\sin^3(\alpha) \\ \qquad\qquad + C_n^5\cos^{n-5}(\alpha)\sin^5(\alpha) - \cdots \\ \cos(n\alpha) = \cos^n(\alpha) - C_n^2\cos^{n-2}(\alpha)\sin^2(\alpha) \\ \qquad\qquad + C_n^4\cos^{n-4}(\alpha)\sin^4(\alpha) \\ \qquad\qquad - C_n^6\cos^{n-6}(\alpha)\sin^6(\alpha) - \cdots. \end{cases} \tag{2}$$

Using an electronic comparator to detect zero crossings, quadrature binary pulses may in turn be obtained from $\sin(n\alpha)$ and $\cos(n\alpha)$. These pulses are more readily decoded using most standard servo controllers or CNC systems for position information. A further four times interpolation can be obtained from these signals. The method eliminates the need for precision analog-to-digital signal acquisition and processing units within the control system for interpolation purposes, since interpolation has been done independently of the controller.

A look-up table will serve as the inferencing engine to provide the signal interpolation (Section IV). Errors in the originating encoder signals can then be directly reflected in the entries of the look-up table without any separate correction mechanisms. These errors will include waveform distortion error, which has not been duly addressed in the literature reported, to the best of our knowledge.

III. Pre-Interpolation Signal Conditioning

Before interpolation, it is important to correct the errors in the originating encoder signals. Commonly encountered errors in the encoder signals are the mean and phase offsets, amplitude difference and waveform distortion. This section will describe how some of these error components can be calibrated and corrected.

Ideally, the quadrature encoder signals (denoted by u_1 and u_2 respectively) are identical sinusoidal signals displaced by a phase of $\pi/2$ with respect to each other

$$\begin{cases} u_1 = A\cos\alpha \\ u_2 = A\sin\alpha. \end{cases} \tag{3}$$

α denotes the instantaneous phase and A denotes the amplitude of the signals.

According to the Heydermann method [1], the more general equations relating the ideal and practical encoder signals are

$$\begin{cases} \overline{u}_1 = u_1 + m_1 \\ \overline{u}_2 = \dfrac{A_1\cos(\alpha - \varepsilon)}{G} + m_2 \end{cases} \tag{4}$$

where m_1 and m_2 are the mean values of the signals and ε is the phase shift. The $\overline{u}_1$ and $\overline{u}_2$ are values obtained from the encoder. $G = A_1/A_2$ and A_1, A_2 are the actual amplitudes of the encoder signals.

Direct simplification of (3) and (4) yields

$$k_1\overline{u}_1^2 + k_2\overline{u}_2^2 + k_3\overline{u}_1\overline{u}_2 + k_4\overline{u}_1 + k_5\overline{u}_2 = 1 \tag{5}$$

where k_i, $i = 1, 2 \cdots 5$, are the constants, and they can be identified online or offline by a least squares fitting routine [1].

From k_i, the offset parameters of the encoder signal may be derived as follows

$$\begin{cases} \varepsilon = \sin^{-1}\left(\dfrac{k_3}{\sqrt{4k_1k_2}}\right) \\ G = \sqrt{\dfrac{k_2}{k_1}} \\ m_1 = \dfrac{2k_2k_4 - k_3k_5}{k_3^2 - 4k_1k_2} \\ m_2 = \dfrac{2k_1k_5 - k_3k_4}{k_3^2 - 4k_1k_2} \\ A_1 = \sqrt{\dfrac{4k_2(1 + k_1m_1^2 + k_2m_2^2 + k_3m_1m_2)}{4k_1k_2 - k_3^2}}. \end{cases} \tag{6}$$

TABLE I
LOOK-UP TABLE BASED ON $\tilde{u}_1$ ONLY

Index	1 … s-1 s	s+1 …2s-1 2s	……	3s+1 …4s-1 4s
Range	0~π/2	π/2~π	……	3π/2~2π
$\tilde{u}_1$	1/s … (s-1)/s 1	1 … (s-1)/s 1/s		-1 … -(s-1)/s –1/s
sin(*16α*)	0.016…-0.649 0.000	0.649…-0.016 0.0	……	0.649…-0.016 0.0
cos(*16α*)	0.999… 0.760 1.000	0.760… 0.999 1.0	……	0.760… 0.999 1.0

Consequently, the corrected and united signals can be obtained as

$$\begin{cases} \tilde{u}_1 = (\overline{u}_1 - m_1)/A_1 \\ \tilde{u}_2 = \dfrac{(\overline{u}_1 - m_1)\sin\varepsilon + G(\overline{u}_2 - m_2)}{A_1 \cos\varepsilon}. \end{cases} \tag{7}$$

It should be noted that this processing is usually done offline on logged encoder information over the entire travel of the actuator. If $m_1, m_2, \varepsilon, A_1$ and A_2vary significantly with time, a recursive least square fitting can be applied to recursively compute these parameters online as they change.

This method has assumed a sinusoidal structure in the encoder signal in the formulation of the least squares estimation algorithm. As a result, it is not able to handle any error due to waveform distortion.

An illustration of the sinusoidal signals with no waveform distortion, before and after correction, using the above method is given in Fig. 1. The correction parameters are

$$\sin\varepsilon = -0.2805, \quad G = 0.8362, \quad m_1 = 3.519 \times 10^{-4}, \\ m_2 = 0.0022, \quad A_1 = 0.533.$$

IV. CONSTRUCTION OF A LOOK-UP TABLE

While $\sin(n\alpha)$ and $\cos(n\alpha)$ can be computed from (2), it is too inefficient to be viable when the encoder signals are to be processed at high speed, especially when n is large. A look-up table can be designed instead for this purpose. The table can output directly the values of $\sin(n\alpha)$ and $\cos(n\alpha)$, given the inputs $\tilde{u}_1 = \sin\alpha$ and $\tilde{u}_2 = \cos\alpha$.

A. Look-Up Table Based on $\tilde{u}_1$ Only

To simplify the inferencing procedure, the values of $\sin(n\alpha)$ and $\cos(n\alpha)$ can be pre-computed and recorded corresponding to pre-determined samples of either $\tilde{u}_1$ or $\tilde{u}_2$, and the sign of the other (for illustration, we will use $\tilde{u}_1$ and the sign of $\tilde{u}_2$ for this purpose). To simplify the addressing of the table, these samples $(\tilde{u}_1)$ are obtained at equal intervals over the entire amplitude range from -1 to 1 (instead of over the entire range of electrical angle over one period). The samples are obtained at $1/s$ interval over this range, and thus there will be a total number of s samples obtained over each quadrant of the sinusoid. There are thus $4s$ samples per period. A large s will result in finer interpolation resolution; however, the trade-off is a larger look-up table and increased sensitivity to noise.

As an example, consider $n = 16$ and $s = 1024$. The look-up table is accordingly set up as in Table I for one period.

Given the real-time value of $\tilde{u}_1$ and sign of $\tilde{u}_2$, the associated table entry can be directly located since the sample interval is fixed and known. Table II serves as the search table to locate the relevant entries efficiently. We first define indices N_s, N_c, N_0 as

$$\begin{cases} N_s = \mathrm{round}(s \times \tilde{u}_1) \\ N_c = \tilde{u}_2 \\ N_0 = s. \end{cases}$$

TABLE II
INDEX TABLE

Condition		n_{index}	Range of α
N_s>0	N_c>0	N_s	0~π/2
	N_c<0	$2N_0$-N_s	π/2~π
N_s<=0	N_c>0	$3N_0$+N_s	3π/2~2π
	N_c<0	$3N_0$-N_s	π~3π/2

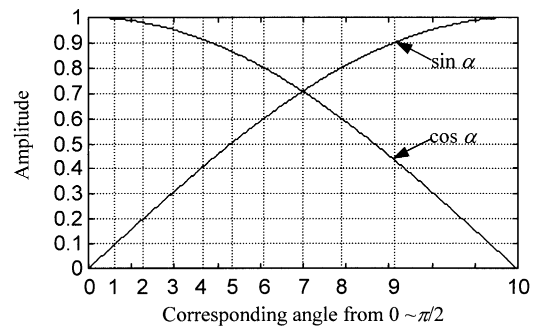

Fig. 2. Variation of amplitude against angle.

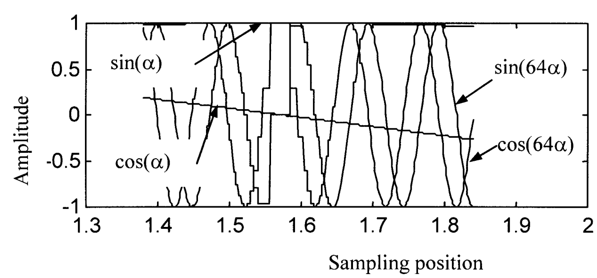

Fig. 3. Interpolation based on $\tilde{u}_1$.

One potential problem with this tabulation method arises due to the large nonlinear variation of the amplitude of $\tilde{u}_1$ with the electrical angle α. Using pre-recorded samples of $\tilde{u}_1$ equally spaced in amplitude, will mean a varying interval of the corresponding angle as shown in Fig. 2.

This angle resolution is poor near the vicinity of $\tilde{u}_1 = \sin\alpha \approx 1$. Thus, to have sufficient information pre-recorded from this part of the signal, s must be very large which will correspondingly imply a large look-up table. Fig. 3 shows the interpolation result, when $s = 5000$ and $n = 64$.

TABLE III
LOOK-UP TABLE FOR $n = 16$

Index	1 … s-1 s	s+1 … 2s	……	7s+1 …… 8s
Range	0~π/4	π/4~π/2	……	7π/4~2π
sin(16α)	0.999 … 1.000	0.999 … 0.000	……	0.000 … 0.000
cos(16α)	0.016 … 0.000	0.023 … 1.000	……	1.000 … 1.000

TABLE IV
INDEX TABLE

Condition		n_{index}	Range of α	$\tilde{u}_1$ or $\tilde{u}_2$ used
N_s>N_0	N_c>0	$2N_0$-N_c	π/4~π/2	$\tilde{u}_2$
	N_c<0	$2N_0$+N_c	π/2~3π/4	
N_s<=-N_0	N_c>0	$6N_0$+N_c	5π/4~3π/2	
	N_c<0	$6N_0$+N_c	3π/2~7π/4	
N_c>N_0	N_s>0	N_s	0~π/4	$\tilde{u}_1$
	N_s<0	$8N_0$+N_s	7π/4~2π	
N_c<=-N_0	N_s>0	$4N_0$-N_s	3π/4~π	
	N_s<0	$4N_0$+N_s	π~5π/4	

The waveforms of $\sin(64\alpha)$ and $\cos(64\alpha)$ are distorted around $\sin(\alpha) \approx 1$.

B. Look-Up Table Based on Both $\tilde{u}_1$ and $\tilde{u}_2$

To overcome this difficulty, amplitudes of both $\sin(n\alpha)$ and $\cos(n\alpha)$ may be pre-recorded, since for the region around $\sin(\alpha) \approx 1$, $\tilde{u}_2 = \cos(\alpha)$ has a much more even relationship between the amplitude and phase angle. Therefore, $\tilde{u}_2$ can be used more effectively for the inferencing procedure instead in these areas. To this end, we propose that for $|\tilde{u}_1| < 0.707$, we will use $\tilde{u}_1$ as the basis to search for the table entry. Otherwise, the amplitude of $\tilde{u}_2$ is used. Essentially, this means the look-up table now consists of more portions (eight instead of four) corresponding to various parts of $\tilde{u}_1$ and $\tilde{u}_2$.

The look-up table for $n = 16$ is given in Table III.

There are s samples over the amplitude range of 0 to 0.707 and, therefore, the division is now $0.707/s$. There are also s samples over the range 0.707 to 1, thus having a finer division in this poor angle resolution region. To facilitate the efficient and quick access to the appropriate part of the look-up table, an index table (similar to Table II) is useful. To this end, we also define indices N_s, N_c, N_0 as:

$$\begin{cases} N_s = \text{round}\left(\dfrac{s}{\sin(\pi/4)} \times \tilde{u}_1\right) \\ N_c = \text{round}\left(\dfrac{s}{\sin(\pi/4)} \times \tilde{u}_2\right) \\ N_0 = \text{round}(s \times \sin(\pi/4)). \end{cases} \tag{8}$$

Based on these indices, the following index table (Table IV) yields the actual points where the appropriate $\sin(n\alpha)$ and $\cos(n\alpha)$ can be directly located (n_{index}), corresponding to the various parts of $\tilde{u}_1$ and $\tilde{u}_2$ respectively.

Fig. 4 shows the interpolation results when $s = 707$ and $n = 64$. There is no waveform distortion even though $s = 707$, which is smaller than that used in Fig. 2 ($s = 5000$).

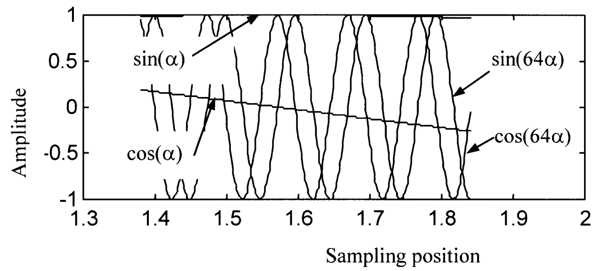

Fig. 4. Interpolation based on both $\tilde{u}_1$ and $\tilde{u}_2$.

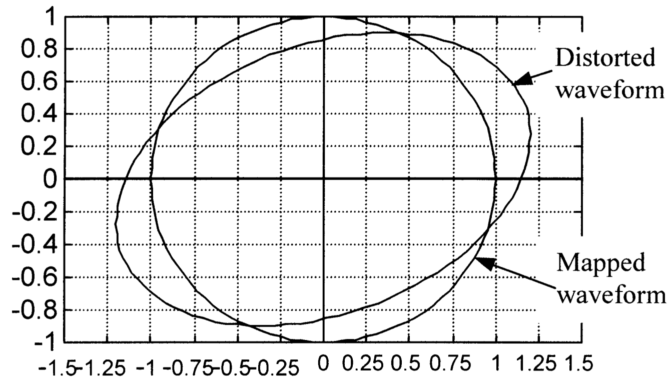

Fig. 5. Waveform error mapping.

C. Maximum Interpolation

The maximum interpolation n^* achievable is limited by the minimum number of samples to be recorded in one period of the raw encoder sinusoid signal, and the minimum number of samples required to appear over one period of the high-order sinusoid to be generated according to the following equation:

$$n^* = \frac{4s_1}{s_2}. \tag{9}$$

Here, s_1 is the maximum number of samples recorded in one period of $\sin(\alpha)$, and s_2 is the minimum number of samples to appear over one period of $\sin(n\alpha)$ [since one cycle of $\sin(\alpha)$ will contain n cycles of $\sin(n\alpha)$]. The factor of four arises due to the additional fourfold interpolation for digital encoders. For example, if $s_2 = 6$ and an interpolation of 1024 is required (i.e., $n^* = 1024$), then $s_1 = 1536$, i.e., we need to acquire at least 1536 samples over one period of the raw signal. It should be noted that the limit on interpolation due to sampling is considered in (9). Noise sensitivity is another issue that is considered separately in Section IV-G.

D. Waveform Distortion

In Section III, we have assumed that the signals from the encoder are ideal and periodic sinusoidal signals, with no waveform distortion. In practice, the waveform of the actual encoder

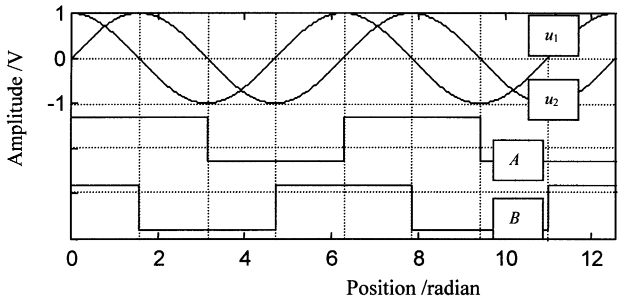

Fig. 6. Quadrature sinusoidal signal decoding.

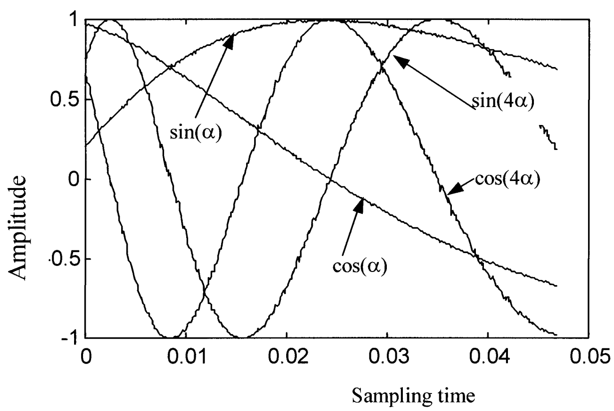

Fig. 7. Interpolation $(n = 4)$.

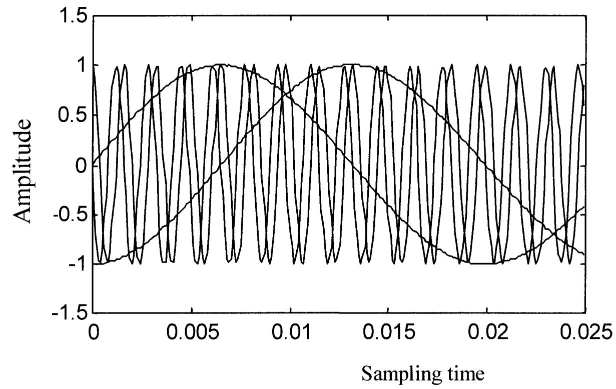

Fig. 8. Interpolation $(n = 16)$.

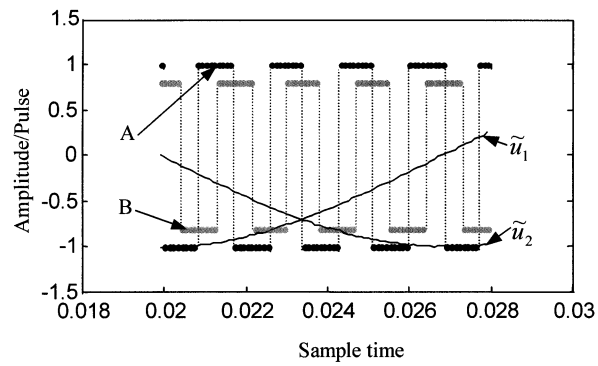

Fig. 9. Interpolation and conversion to quadrature pulses $(n = 16)$.

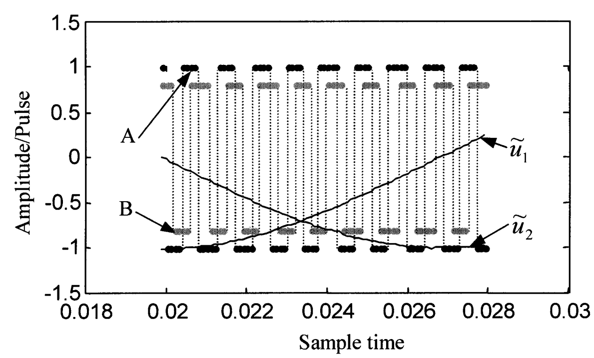

Fig. 10. Interpolation and conversion to quadrature pulses $(n = 32)$.

signals deviate from the ideal sinusoidal waveform. Therefore, corrections based on the ideal sinusoidal waveform assumption may yield inaccurate position information that may not be acceptable for applications with high-precision requirements. It is more reasonable to assume that the encoder signal is periodic and reproducible in the waveform that is not necessarily sinusoidal. In this case, since the nonsinusoidal waveforms are available, we can use an error mapping method to map them into sinusoidal ones. The idea is depicted in Fig. 5. The look-up tables of Section IV continue to be applicable.

It should be cautioned that this is possible if the distorted waveforms are periodic and there exists a one-to-one mapping of each point to the ideal sinusoid. It is also necessary for the A/D converter to have a wordlength sufficient to resolve two different points on the waveform.

E. Conversion to Binary Pulses

In order for the encoder signals to be received by a general-purpose incremental encoder interface, the quadrature sinusoidal signals must be converted to a series of binary pulses. An analog comparator may be used to transform the high-order sinusoids into pulses. As shown in Fig. 6, the comparator will simply switch the pulse signals when the associated sinusoidal signal crosses zero. The rest of the analog information will not be used.

Alternatively, this transformation can be more efficiently done within the look-up table. The $\sin(n\alpha)$ and $\cos(n\alpha)$ entries in the table can be directly converted into binary values (A and B respectively) according to the following equations:

$$\begin{cases} A = 1, & \sin(n\alpha) \geq \delta \\ \quad = -1, & \sin(n\alpha) \leq -\delta \end{cases}$$
$$\begin{cases} B = 1, & \cos(n\alpha) \geq \delta \\ \quad = -1, & \cos(n\alpha) \leq -\delta. \end{cases} \tag{10}$$

Thus, we can generate A and B which are quadrature square curves directly from Table III. δ can be 0 or a small value set according to the threshold of measurement noise.

F. Direct Conversion to Digital Position

The pulse information in Table III can be easily converted into digital position values, which can be directly used for control purposes without further computations. This is especially true

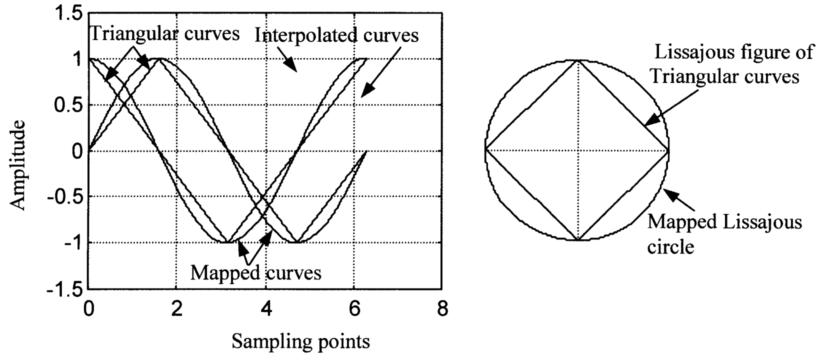

Fig. 11. Mapped interpolation ($n = 8$).

if the aforementioned interpolation method is integrated into a general digital controller. Alternatively, the encoder card can be made PC-bus based and the general motion controller can acquire the digital position value directly from the register or shared memory. In this case, the D/A converters are not required.

G. Practical Constraints

Similar to existing interpolation methods based on computation of the electrical angle, the proposed approach is also subject to practical constraints such as noise sensitivity and digitization errors.

In order to resolve each sample (separated by $1/s^*$) of the encoder's signal, the A/D converter should have a wordlength n sufficient to meet the following condition:

$$2^{n-1} > \frac{s^*}{2}.$$

Conversely, given a fixed wordlength n, the resolution and therefore the final interpolation achievable will be limited accordingly.

The bandwidth (B) of the control electronics limits the number of pulses which can be acquired per unit time, which in turn limits the maximum velocity $(V_{\max})$ achievable by the actuator in order for interpolation at n^* to still work well. An estimate of the velocity can be obtained from the following equation, where e_p is the encoder pitch:

$$V_{\max} = \frac{B.e_p}{n^*}.$$

Noise arising in encoder signals should be minimized prior to interpolation by proper shielding and grounding of the transmission and reception circuits. However, it is unlikely that it can be totally eliminated. The higher order sinusoids generated from the proposed interpolation approach can be contaminated by measurement noise. However, the final measurement can be relatively unaffected if the conversion to binary pulses at the zero crossing is properly handled to avoid erroneous switching due to noise. This is usually handled in practice via the hysteresis (or switching threshold) in the comparator so that switching can only happen when the zero point is crossed sufficiently. The hysteresis level is selected to correspond to an estimate of the amplitude of the measurement noise.

V. Experiments

A dSPACE controller with a high-speed A/D card (with a wordlength of 12 b) is first used to acquire the raw quadrature sinusoidal signals from the Heidenhein linear encoder LIP481 for the pre-interpolation signal conditioning. The compensation parameters are: $m_1 = -0.0126$, $m_2 = 1.4483e - 004$, $A_1 = 0.1331$, $A_2 = 0.1221$. This process is carried out offline.

Interpolation is subsequently carried out based on the proposed method. Fig. 7 shows the interpolation result with $n = 4$. This (as well as subsequent results to be presented) is done online with the actuator controlled to run at a constant speed.

Fig. 8 shows the results with $n = 16$.

Fig. 9 shows the results with $n = 16$ and in addition, the look-up table entries are converted to binary values, according to Section IV-D, to yield binary pulses directly. To allow the pulses (with similar amplitudes) to be shown more clearly in Fig. 9, the amplitude of B is deliberately set to 0.8.

Fig. 10 shows the results with $n = 32$.

To more clearly illustrate the situation with nonsinusoidal encoder signals and the correction using mapping, triangular waveforms are simulated and mapped to sinusoidal ones. The interpolation results for $n = 8$ and their Lissajous figures are shown in Fig. 11. Current interpolators that rely on a computation of electrical angle for interpolation will be inadequate when applied to these periodic but nonsinusoidal signals.

VI. Conclusion

A new interpolation method is developed in this paper, suitable for increasing the measurement resolution obtainable from quadrature encoder signals. Based on the existing sinusoidal signals, high-order sinusoids are derived, from which binary pulses are generated which can be decoded using only standard servo controllers for position information. A look-up table, constructed off-line, serves as the inferencing engine for an effective way of generating the signals online with minimal computational burden. Imperfections in the encoder signals can be directly compensated offline in the look-up table, including mean and phase offsets, amplitude difference and waveform distortion. Simulation and experimental results provided illustrate the effectiveness of the proposed method.

REFERENCES

[1] P. L. M. Heydemann, "Determination and correction of quadrature fringe measurement errors in interferometers," *Appl. Opt.*, vol. 20, no. 19, Oct. 1981.

[2] K. P. Birch, "Optical fringe subdivision with nanometric accuracy," *Precision Eng.*, vol. 12, no. 4, Oct. 1990.

[3] K. Watanabe and H. Yokote, "A microstep controller of a DC servomotor," *IEEE Trans. Instrum. Meas.*, vol. 39, pp. 60–62, Dec. 1990.

[4] N. Hagiwara and H. Murase, "A method of improving the resolution and accuracy of rotary encoders using a code compensation technique," *IEEE Trans. Instrum. Meas.*, vol. 41, pp. 98–101, Feb. 1992.

[5] H. Yokote and K. Watanabe, "A hybrid digital and analog controller for DC and brushless servomotors," *IEEE Trans. Instrum. Meas.*, vol. 39, pp. 259–263, Apr. 1989.

[6] J. R. R Mayer, "High resolution of rotary encoder analog quadrature signals," *IEEE Trans. Instrum. Meas.*, vol. 43, pp. 494–498, June 1994.

[7] J. S. Chen and C. C. Liang, "Improving the machine accuracy through machine tool metrology and error correction," *Int. J. Adv. Manufact. Technol.*, vol. 11, 1996.

[8] J. Ni, "CNC machine accuracy enhancement through real time error compensation," *J. Manufact. Sci. Eng.*, vol. 119, Nov. 1997.

[9] G. C. Goodwin and R. L. Payne, *Dynamic System Identification: Experiment Design and Data Analysis*. New York: Academic, 1977.

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